2018 ECOmp 研究部会
February 28, 2018, Fukuoka, Japan

Estimated Convergence Points Applied to Evolutionary Computation

Jun YU
Graduate School of Design
Kyushu University, Fukuoka, Japan
yujun@kyudai.jp

Hideyuki TAKAGI
Faculty of Design
Kyushu University, Fukuoka, Japan
http://www.design.kyushu-u.ac.jp/~takagi/

I. Estimation Method

An estimated convergence point can be calculated from the moving vectors between parent individuals and their offspring [1]. Let $\boldsymbol{a}_i$ and $\boldsymbol{c}_i$ in the Figure 1 be the i-th parent individual and its offspring, respectively ($\boldsymbol{a}_i, \boldsymbol{c}_i \in \mathbb{R}^d$). Then, the i-th moving vector is defined as a direction vector, $\boldsymbol{b}_i = \boldsymbol{c}_i - \boldsymbol{a}_i$. Its unit direction vector is given as $\boldsymbol{b}_{0i} = \boldsymbol{b}_i/||\boldsymbol{b}_i||$, i.e. $\boldsymbol{b}_{0i}^T \boldsymbol{b}_{0i} = 1$.

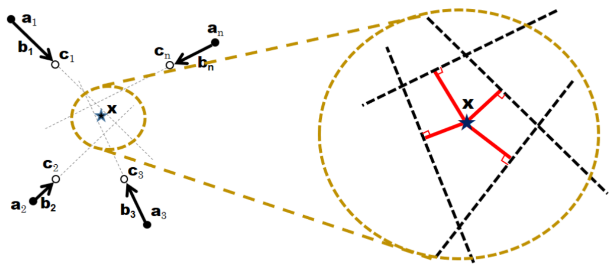

Fig. 1: Moving vector $\boldsymbol{b}_i$ ($= \boldsymbol{c}_i - \boldsymbol{a}_i$) is calculated from a parent individual $\boldsymbol{a}_i$ and its offspring $\boldsymbol{c}_i$ in the d-dimensional searching space. The $\star$ mark is the convergence point for these vectors.

Let $\boldsymbol{x} \in \mathbb{R}^d$ be the point that is the nearest to the n extended directional line segments, $\boldsymbol{a}_i + t_i \boldsymbol{b}_i$ ($t_i \in R$). By *nearest*, we mean that the total distance from $\boldsymbol{x}$ to the n extended directional line segments, $\mathrm{J}(\boldsymbol{x}, \{t_i\})$ in Eq.(1), becomes the minimum.

As the minimum line segment from the convergence point $\boldsymbol{x}$ to the extended directional line segments is the orthogonal projection from $\boldsymbol{x}$, we can remove t_i by inserting an orthogonal condition, the Eq. (2), into the Eq. (1).

$$\mathrm{J}(\boldsymbol{x}, \{t_i\}) = \sum_{i=1}^{n} \|\boldsymbol{a}_i + t_i \boldsymbol{b}_i - \boldsymbol{x}\|^2 \quad (1)$$

$$\boldsymbol{b}_i^{\mathrm{T}}(\boldsymbol{a}_i + t_i \boldsymbol{b}_i - \boldsymbol{x}) = 0 \quad \text{(orthogonal condition)} \quad (2)$$

The $\hat{\boldsymbol{x}}$ that minimizes the total distance in the Eq. (1) is obtained by partially differentiating each element of $\boldsymbol{x}$ and setting them equal 0. Finally, the convergence point $\hat{\boldsymbol{x}}$ is given by Eq. (3), where $\boldsymbol{I}_d$ is a unit matrix.

$$\hat{\boldsymbol{x}} = \left\{ \sum_{i=1}^{n} \left(\boldsymbol{I}_d - \boldsymbol{b}_{0i} \boldsymbol{b}_{0i}^{\mathrm{T}} \right) \right\}^{-1} \left\{ \sum_{i=1}^{n} \left(\boldsymbol{I}_d - \boldsymbol{b}_{0i} \boldsymbol{b}_{0i}^{\mathrm{T}} \right) \boldsymbol{a}_i \right\} \quad (3)$$

II. Application of the Estimated Convergence Point for Evolutionary Computation

A. The Simplest Application

The simplest use of the estimated convergence point is to use it as a new elite individual and replace the worst individual. When the estimated convergence point locates near the global optimum as we expect, we can expect that the elite individual becomes a powerful parent. When it is not close to the global optimum unfortunately, the elite may not be remarkable but is still better than the worst individual replaced by it. We can say that it is a *low risk, high return* stragegy.

B. Weight-based Estimation Method

Although the above basic method for estimating the convergence point handles all moving vectors fairly, all moving vectors do not aim the global optimum directly but just toward *better* directions. As the efficiency of moving vectors is expressed by how fitness is improved, we may be able to improve the precision of the estimated convergence point by weighting moving vectors with their efficiency. We introduced weights into the basic estimation method and develop its improved version [2].

C. Application to Multimodal Optimization

The basic estimation method is surely effective for unimodal tasks [1], but it is not always true for multimodal tasks because moving vectors go towards different local optima. To make this method applicable to general optimization tasks, we must cluster moving vectors according to their directions to local minima first and then apply the method to estimate each local minimum. We proposed a cluttering method for bipolar tasks [3] and then extend it for generic multimodal tasks [4].

Acknowledgment

This work was supported in part by Grant-in-Aid for Scientific Research (JP15K00340).

References

[1] Murata, N., Nishii, R., Takagi, H., and Pei Y., "Analytical Estimation of the Convergence Point of Populations," 2015 IEEE Congress on Evolutionary Computation (CEC2015), pp. 2619–2624, Sendai, Japan.

[2] 余俊，高木英行「信頼性重みを導入した個体群の収束点推定精度の向上」第 13 回進化計算学会研究会, pp.59-63, 草津.

[3] Yu J. and Takagi H., "Clustering of Moving Vectors for Evolutionary Computation," 7th Int. Conf. on Soft Computing and Pattern Recognition (SoCPaR2015), pp.169-174, Fukuoka, Japan.

[4] 余俊，高木英行「個体間距離順位とフィットネス順位に基づく局所解領域の推定」第 12 回進化計算学会研究会プログラム，pp.203-206，福岡.