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REAL-TIME RESTORATION OF BIOMEDICAL SIGNALS UNDER ADDITIVE NOISE

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Abstract - **In this paper we shall examine the real-time restoration of biomedical signals under additive noise. The observation of these signals is defined by a mathematical model. We proposed a couple of restoration filters, composed of a series of the this model. These filters restored band-limited approximations to their original signals in real-time. We introduced the redundancy into the restoration filter suppress the additive noise.**

I. Introduction

During clinical analysis or diagnosis, nonstationary biomedical signals, commonly referred to as the patient's vital signs, are measured. However, the instruments involved, such as catheter manometers, ambulatory electrocardiographs and thermo-dilution sensors, are themselves susceptible to distortion and noise. We therefore need to be able to restore signals to their original state. A couple of restoration filters, composed of the observation model, were proposed [1]-[6]. These filters restored band-limited approximations to their original signals in real-time. However, the noise included in observed signals has never been discussed. We therefore consider to suppress the additive noise by introducing the redundancy into the proposed filter. In this study, the theory behind, and application of, a real-time restoration are discussed.

II. Methods

A. Definition of Problem

We treat signals in $L_2[-\infty, \infty]$. Assume f to be an original signal in L_2, and A to be the observed operator from L_2 to L_2. The observed signal, a_0, is given by

$$a_0 = Af + n, \tag{1}$$

where n is the additive noise. Assuming B to be the restoration filter and P to be the band-limitation filter, a restored band-limited signal, Pf, is defined by

$$Pf = Ba_0. \tag{2}$$

We consider the following limitation, in the restoration process in (2):

(i) Pf is nearly the same as f.
(ii) real-time restoration.

To satisfy (ii), we restore the signal by using a signal observed in the past. Assume A in (1) to be

$$\Gamma(s)f(t) = \frac{1}{s}\int_0^\infty \exp\left(-\frac{\tau}{s}\right) f(t-\tau)d\tau, \quad s > 0. \tag{3}$$

The transfer functions of many biomedical instruments are expressed by the combination of $\Gamma(s)$. $\Gamma(s)$ corresponds to the first-order low-pass filter with a time constant of s. Equation (3) merely requires a signal from the past to the present. Moreover, because the signal from the past decays exponentially, the signal in the present time is emphasized in (3). In this manner, $\Gamma(s)$ naturally expresses the present-time signal.

B. Band-Limited Restoration

In this section, we consider the band-limited restoration of the signal. The observed systems are represented not only by the first-order system in (3), but also by the higher-order system. We define the *Lth*-order observed operator as

$$A = \prod_{l=1}^{L} \Gamma(s_l), \quad \mathrm{Re}\{s_l\} > 0 \tag{4}$$

and the *Mth*-order band-limited operator as

$$P = \Gamma(s_0)^M, \quad s_0 < \mathrm{Re}\{s_l\}, \tag{5}$$

where $\{s_l\}$ are any complex values. To satisfy (i) in section A, s_0 must be set to a small value.

We propose a restoration filter, which is composed of the band-limited operator, $\Gamma(s_0)$. The fundamental observation signals, $\{a_m\}$, are successively derived by

$$a_m = \{I - \Gamma(s_0)\}a_{m-1}, \quad m = 1, 2, \ldots, . \tag{6}$$

By using $\{a_m\}$, we derive the order, M, and the coefficients, $\{b_m\}$, which satisfy

$$Pf = \sum_{m=0}^{M} b_m a_m. \tag{7}$$

The restoration filter for (4) is derived by

$$B = \sum_{m=0}^{M} b_m \{I - \Gamma(s_0)\}^m . \tag{8}$$

Equation (8) is composed of $\Gamma(s_0)$, which requires no future-time signal. Thus, the restoration filter, B, satisfies (ii) in section A.

C. *Restoration Filter* ($M = L$) [4]-[6]

To satisfy (7), the order of the restoration filter, M, must be more than the order of the observation system, L. We reported the restoration filter in the case of M=L, that was derived by

$$B = \sum_{m=0}^{L} b_m \{I - \Gamma(s_0)\}^m \tag{9}$$

where

$$b_m = \sum_{i=0}^{\binom{L}{m}} \prod_{j=1}^{m} \frac{s_{d(i,j)} - s_0}{s_0}. \tag{10}$$

$\binom{L}{m}$ is a binomial coefficient and $d(i,j)$ is the natural number that satisfies $1 \le d(i,1) < \ldots < d(i,m) \le L$ and $\{d(i',1), \ldots, d(i',m)\} \ne \{d(i'',1), \ldots, d(i'',m)\}$ while $i' \ne i''$. In this case, we did not consider the influence of noise.

D. *Redundant Restoration Filter* ($M > L$)

In this section, we try to suppress the noise by introducing redundancy into the order of the restoration filter. That is, we propose M (> L)*th*-order restoration filter, that restore the signals observed by the *Lth*-order system.

The restoration filter for first-order observation system is derived by

$$B = \sum_{m=0}^{M} b_m \{I - \Gamma(s_0)\}^m \tag{11}$$

where

$$b_m = (-1)^m \left\{ \binom{M-1}{m} - \binom{M-1}{m-1} \frac{s_1 - s_0}{s_0} \right\}. \tag{12}$$

The restoration filter for the *Lth*-order observation system is derived by

$$B = \sum_{m=0}^{M} b_m \{I - \Gamma(s_0)\}^m \tag{13}$$

where

$$b_m = (-1)^m \sum_{l=0}^{L} (-1)^l \binom{M-L}{m-l} \sum_{i=0}^{\binom{L}{l}} \prod_{j=1}^{l} \frac{s_{d(i,j)} - s_0}{s_0}. \tag{14}$$

Fig. 1 shows a block diagram of the *Mth*-order restoration filter for an *Lth*-order observation system.

E. *Parameter Estimation under additive noise*

To realize the restoration, B, parameters, such as s_0, $\{b_m\}$ and M, are decided in advance.

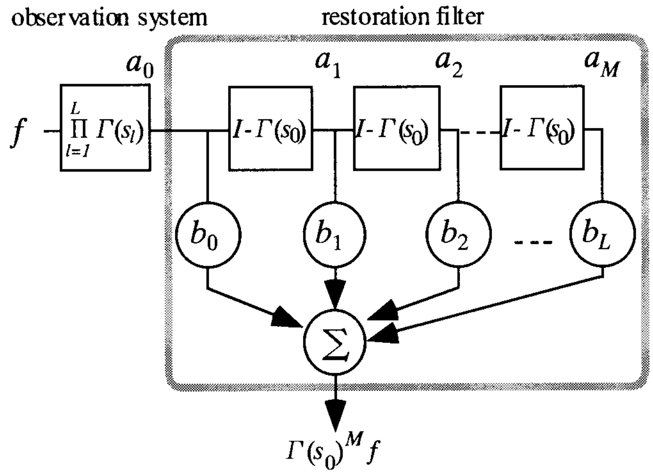

Fig. 1. *Mth*-order Restoration filter for *Lth*-order observation system.

The parameter of the band-limited operator, s_0, affects the precision of the restoration. The band-limited signal, Pf, approaches f by setting s_0 at as small a value as possible, in order to satisfy

$$20 \log |P(\omega_c)| \ge -3[dB], \tag{15}$$

where ω_c is the maximum angular frequency of the original signal and $P(\omega)$ is the frequency characteristics of P.

When the parameters of the observed operator, $\{s_l\}$, are unknown, $\{b_m\}$ are calculated by using an already-known step signal as f. $\{b_m\}$ are obtained by satisfying

$$\sum_{m=0}^{M} b_m \{I - \Gamma(s_0)\}^m a_0 = Pf . \tag{16}$$

To decide the order of the restoration filter, M, we employ evaluation functions that indicate the precision of restoration. At first, we consider two types of relative errors. One is the error between a band-limited signal and a restored signal,

$$J_1 = \|Pf - Ba_0\| / \|Pf\|, \tag{17}$$

which expresses the power of noise. The other is the error between an original signal and a restored signal,

$$J_2 = \|f - Ba_0\| / \|f\|, \tag{18}$$

which expresses the distortion of the signal. J_1 is inverse proportional to the order of the restoration filter while J_2 is proportional to that. Thus, we proposed new evaluation function,

$$J_3 = J_1(M)/J_1(L) + J_2(M)/J_2(L), \tag{19}$$

to decide the order of the restoration filter. J_3 evaluates both shape and noise. The order, M, minimizing J_3 becomes the optimum order for restoration.

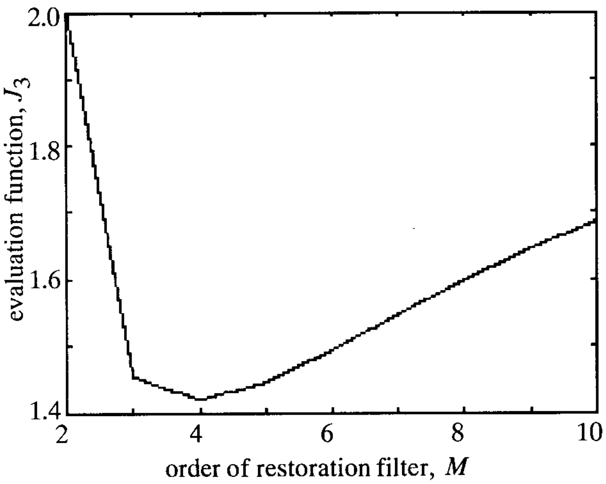

Fig. 2. Optimum order of the restoration filter. (L=2, s_1, s_2=0.03±j0.04, s_0=0.005, σ=10^{-8}).

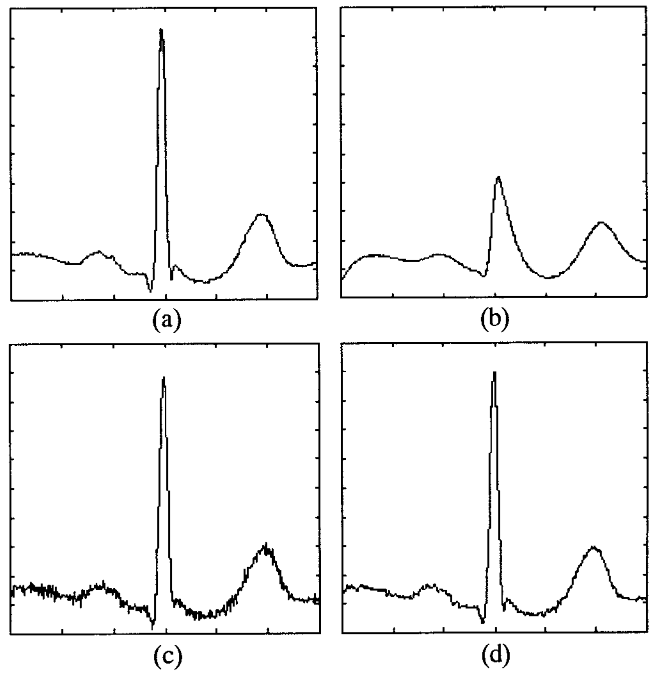

Fig. 3. (a) Original ECG signal, f, (b) observed signal , a_0, by ambulatory ECG system (L=2), (c) restored signal (M=2), and (d) restored signal (M=4).

III. Simulation

We confirmed restorative capabilities of the proposed filter, by using the step signal. Fig. 2 shows the order of restoration filter versus the value obtained by the evaluation function, J_3. In this case, the optimum order was fourth.

The theory was applied to the ambulatory electrocardiogram system (Fig. 3). We assumed that the observation system was second-order. White noise was added to the original ECG signal. When the order of restoration filter was equal to that of the observation system, the restored signal was noisy (Fig.3 (c)). On the other hand, when the order was set to forth, the noise of the signal was suppressed (Fig.3 (d)) and the signal was roughly equal to the original one.

IV. Conclusion

Optimum restoration filters under additive noise were proposed. These filters, composed of redundant linear combination of fundamental filters, restored band-limited approximations to their original signals in real-time. These filters were applicable, not only to first-order measurement systems, but also to those of a higher order. Furthermore, they were applicable to both high and low-pass measurement systems. We have determined that this method will be useful in a broad range of practical situations.

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