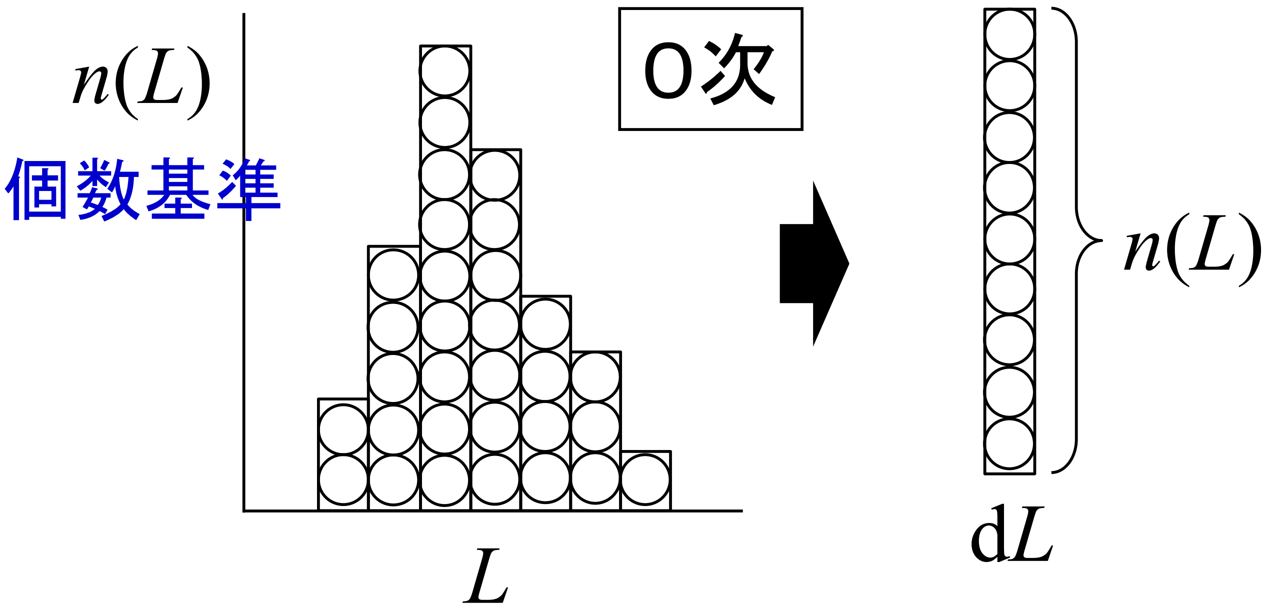
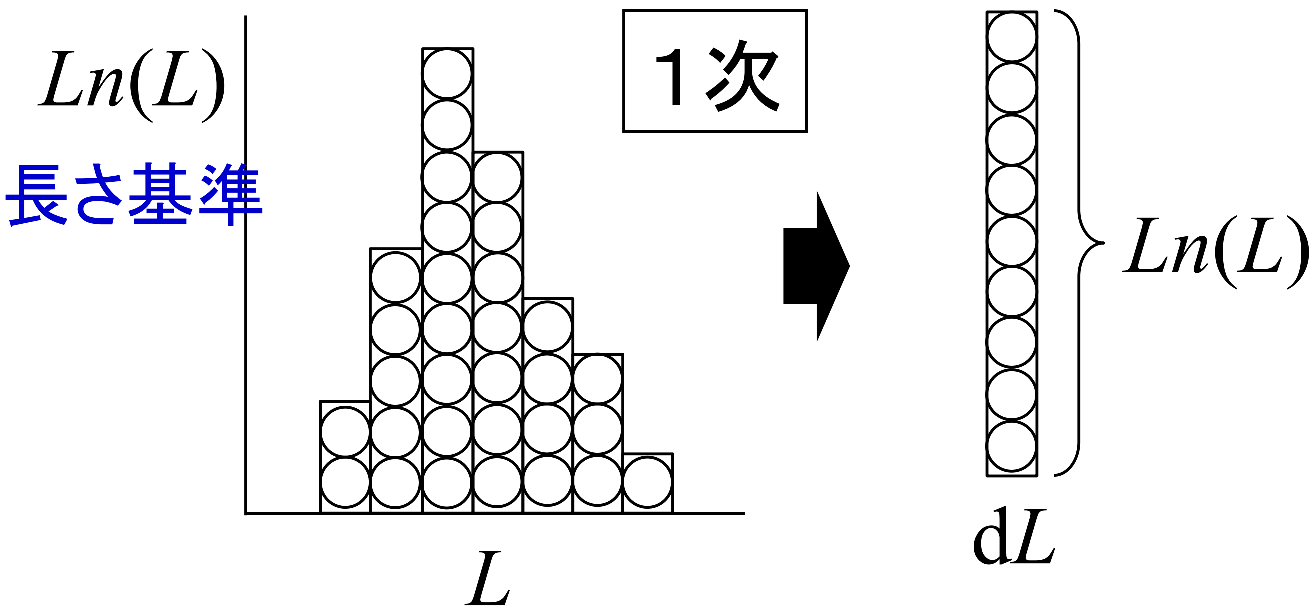
分布関数のモーメント



$$n(L)\times \mathrm{d}L=\mathrm{d}N_{\mathrm{T}}$$

$$N_{\mathrm{T}}=\int_0^{\infty} n(L)dL$$

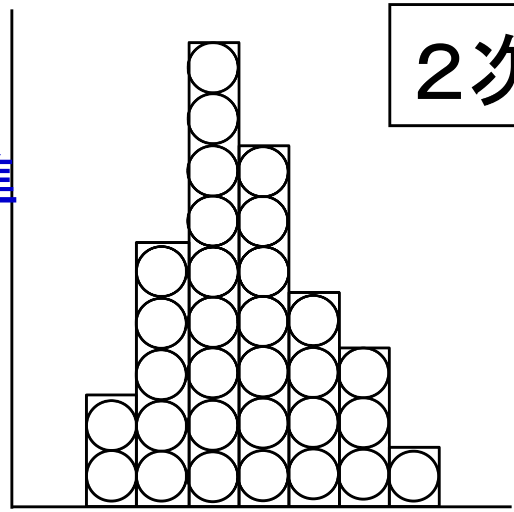


$$Ln(L)\times \mathrm{d}L=\mathrm{d}L_{\mathrm{T}}$$

$$L_{\mathrm{T}}=\int_0^{\infty} Ln(L)dL$$

$\phi_S L^2 n(L)$

表面積基準



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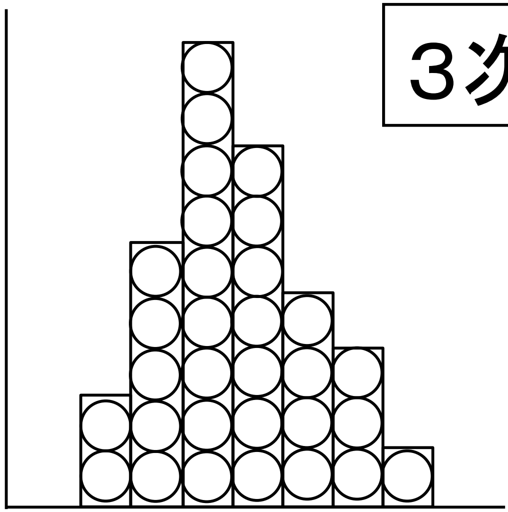
$\phi_S L^2 n(L) \times dL = dA_T$

$$A_T = \phi_S \int_0^\infty L^2 n(L) dL$$

$\phi_V L^3 n(L)$

体積基準



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$$V_T = \phi_V \int_0^\infty L^3 n(L) dL$$

$$M_T = \rho_c \phi_V \int_0^\infty L^3 n(L) dL$$

一般式

$$m_j = \int_0^\infty L^j n(L) dL$$

j=0(個数), 1(長さ), 2(表面積), 3(体積または質量)

（ア）０次モーメントの計算

$$m_0 = \int_0^\infty n(L)\mathrm{d}L$$

$$m_0 = \int_0^\infty n_0 \exp\left(-\frac{L}{G\tau}\right)\mathrm{d}L \quad \left[n(L) \equiv n_0 \exp\left(-\frac{L}{G\tau}\right)\right]$$

$$m_0 = \int_0^\infty n_0 \exp(-X)(G\tau)\mathrm{d}X$$

変数変換$(X=L/G\tau)$

$$\left[\frac{\mathrm{d}X}{\mathrm{d}L} = \frac{\mathrm{d}}{\mathrm{d}L}\frac{L}{G\tau} = \frac{1}{G\tau} \to \mathrm{d}L = (G\tau)\mathrm{d}X\right]$$

$$m_0 = n_0 G\tau \int_0^\infty \exp(-X)\mathrm{d}X$$

$\to 1$

$$m_0 = n_0 G\tau\left[-\exp(-X)\right]_0^\infty$$

$$\boxed{m_0 = n_0 G\tau}$$

(イ) 1次モーメントの計算

$$m_1 = \int_0^\infty L n(L) \mathrm{d}L$$

$$m_1 = \int_0^\infty L n_0 \exp\left(-\frac{L}{G\tau}\right) \mathrm{d}L \quad \left[n(L) \equiv n_0 \exp\left(-\frac{L}{G\tau}\right) \right]$$

$$m_1 = \int_0^\infty (G\tau X) n_0 \exp(-X)(G\tau) \mathrm{d}X$$

$$\left[\begin{array}{l} X \equiv L/G\tau ,\ L = G\tau X \\ \dfrac{\mathrm{d}X}{\mathrm{d}L} = \dfrac{\mathrm{d}}{\mathrm{d}L}\dfrac{L}{G\tau} = \dfrac{1}{G\tau} \\ \mathrm{d}L = (G\tau)\mathrm{d}X \end{array} \right]$$

$$m_1 = n_0 (G\tau)^2 \int_0^\infty X \exp(-X) \mathrm{d}X$$

$$m_1 = n_0 (G\tau)^2 \left[\underbrace{[-X \exp(-X)]_0^\infty}_{\to 0} - \underbrace{\int_0^\infty 1 \cdot [-\exp(-X)] \mathrm{d}X}_{\to (-1)} \right]$$

$$\boxed{m_1 = n_0 (G\tau)^2}$$

部分積分法 $f(X)=X,\ g'(X)=\exp(-X)$ と置く

$$\left[\int_0^\infty f(X) g'(X) \mathrm{d}X = [f(X)g(X)]_0^\infty - \int_0^\infty f'(X) g(X) \mathrm{d}X \right]$$

(ウ)２次モーメントの計算

$$m_2 = \int_0^\infty L^2 n(L)\mathrm{d}L$$

$$m_2 = \int_0^\infty L^2 n_0 \exp\left(-\frac{L}{G\tau}\right)\mathrm{d}L \quad \left[n(L) \equiv n_0 \exp\left(-\frac{L}{G\tau}\right)\right]$$

$$m_2 = n_0 (G\tau)^3 \int_0^\infty X^2 \exp(-X)\mathrm{d}X \quad \left[X \equiv L/G\tau, L = G\tau X, \mathrm{d}L = (G\tau)\mathrm{d}X\right]$$

$$m_2 = n_0 (G\tau)^3 \left[\underbrace{\left[-X^2 \exp(-X)\right]_0^\infty}_{\to 0} - \int_0^\infty 2X[-\exp(-X)]\mathrm{d}X\right]$$

部分積分法 $f(X)=X^2$, $g'(X)=\exp(-X)$ と置く

$$\left[\int_0^\infty f(X)g'(X)\mathrm{d}X = \left[f(X)g(X)\right]_0^\infty - \int_0^\infty f'(X)g(X)\mathrm{d}X\right]$$

$$m_2 = n_0 (G\tau)^3 \int_0^\infty 2X \exp(-X)\mathrm{d}X$$

$$\boxed{m_2 = 2n_0 (G\tau)^3} \quad \left[\int_0^\infty X\exp(-X)\mathrm{d}X = 1\right]$$

（エ）3次モーメントの計算

$$m_3 = \int_0^\infty L^3 n_0 \exp\left(-\frac{L}{G\tau}\right) \mathrm{d}L \quad \left[n(L) \equiv n_0 \exp\left(-\frac{L}{G\tau}\right) \right]$$

$$m_3 = n_0 (G\tau)^4 \int_0^\infty X^3 \exp(-X) \mathrm{d}X \quad \left[X \equiv L/G\tau ,\, L = G\tau X ,\, \mathrm{d}L = (G\tau)\mathrm{d}X \right]$$

$$m_3 = n_0 (G\tau)^4 \left[\underbrace{\left[-X^3 \exp(-X) \right]_0^\infty}_{\to 0} - \int_0^\infty 3X^2 [-\exp(-X)] \mathrm{d}X \right]$$

部分積分法　$f(X)=X^3$, $g'(X)=\exp(-X)$ と置く

$$m_3 = n_0 (G\tau)^4 \int_0^\infty 6X \exp(-X) \mathrm{d}X$$

$$\left[\int_0^\infty X^2 [-\exp(-X)] \mathrm{d}X = \int_0^\infty 2X \exp(-X) \mathrm{d}X \right]$$

$$\left[\int_0^\infty f(X) g'(X) \mathrm{d}X = \left[f(X) g(X) \right]_0^\infty - \int_0^\infty f'(X) g(X) \mathrm{d}X \right]$$

$$\boxed{m_3 = 6 n_0 (G\tau)^4} \quad \left[\int_0^\infty X \exp(-X) \mathrm{d}X = 1 \right]$$

(オ)4次モーメントの計算

$$m_4 = \int_0^\infty L^4 n_0 \exp\left(-\frac{L}{G\tau}\right) \mathrm{d}L \quad \left[n(L) \equiv n_0 \exp\left(-\frac{L}{G\tau}\right) \right]$$

$$m_4 = n_0 (G\tau)^5 \int_0^\infty X^4 \exp(-X) \mathrm{d}X \quad \left[X \equiv L/G\tau, L = G\tau X, \mathrm{d}L = (G\tau)\mathrm{d}X \right]$$

$$m_4 = n_0 (G\tau)^5 \left[\underbrace{\left[-X^4 \exp(-X) \right]_0^\infty}_{\to 0} - \int_0^\infty 4X^3 [-\exp(-X)] \mathrm{d}X \right]$$

部分積分法 $f(X)=X^4$, $g'(X)=\exp(-X)$ と置く

$$m_4 = n_0 (G\tau)^5 \int_0^\infty 24 X \exp(-X) \mathrm{d}X$$

$$\left[\int_0^\infty X^3 [-\exp(-X)] \mathrm{d}X = \int_0^\infty 6X \exp(-X) \mathrm{d}X \right]$$

$$\left[\int_0^\infty f(X) g'(X) \mathrm{d}X = \left[f(X) g(X) \right]_0^\infty - \int_0^\infty f'(X) g(X) \mathrm{d}X \right]$$

$$\boxed{m_4 = 24 n_0 (G\tau)^5} \quad \left[\int_0^\infty X \exp(-X) \mathrm{d}X = 1 \right]$$

（カ）5次モーメントの計算

$$m_5 = \int_0^\infty L^5 n_0 \exp\left(-\frac{L}{G\tau}\right) \mathrm{d}L \quad \left[n(L) \equiv n_0 \exp\left(-\frac{L}{G\tau}\right)\right]$$

$$m_5 = n_0 (G\tau)^6 \int_0^\infty X^5 \exp(-X) \mathrm{d}X \quad \left[X \equiv L/G\tau, L = G\tau X, \mathrm{d}L = (G\tau)\mathrm{d}X\right]$$

$$m_5 = n_0 (G\tau)^6 \left[\underbrace{\left[-X^5 \exp(-X)\right]_0^\infty}_{\to 0} - \int_0^\infty 5X^4 [-\exp(-X)] \mathrm{d}X \right]$$

部分積分法　$f(X)=X^5$, $g'(X)=\exp(-X)$ と置く

$$m_5 = n_0 (G\tau)^6 \int_0^\infty 120 X \exp(-X) \mathrm{d}X$$

$$\left[\int_0^\infty X^4 [-\exp(-X)] \mathrm{d}X = \int_0^\infty 24 X \exp(-X) \mathrm{d}X\right]$$

$$\left[\int_0^\infty f(X) g'(X) \mathrm{d}X = \left[f(X) g(X)\right]_0^\infty - \int_0^\infty f'(X) g(X) \mathrm{d}X\right]$$

$$\boxed{m_5 = 120 n_0 (G\tau)^6} \quad \left[\int_0^\infty X \exp(-X) \mathrm{d}X = 1\right]$$

一般式

$$m_j = \int_0^\infty L^j n(L)\mathrm{d}L = j!n_0(G\tau)^{j+1}$$

$$N_\mathrm{T} = \int_0^\infty n(L)\mathrm{d}L = m_0 = n_0(G\tau)$$

$$L_\mathrm{T} = \int_0^\infty L n(L)\mathrm{d}L = m_1 = n_0(G\tau)^2$$

$$A_\mathrm{T} = \phi_\mathrm{S}\int_0^\infty L^2 n(L)\mathrm{d}L = \phi_\mathrm{S} m_2 = 2\phi_\mathrm{S} n_0(G\tau)^3$$

$$V_\mathrm{T} = \phi_\mathrm{V}\int_0^\infty L^3 n(L)\mathrm{d}L = \phi_\mathrm{V} m_3 = 6\phi_\mathrm{V} n_0(G\tau)^4$$

$$M_\mathrm{T} = \rho_\mathrm{c}\phi_\mathrm{V}\int_0^\infty L^3 n(L)\mathrm{d}L = \rho_\mathrm{c}\phi_\mathrm{V} m_3 = 6\rho_\mathrm{c}\phi_\mathrm{V} n_0(G\tau)^4$$

平均粒径のモーメント

次数	0	1	2	3	
基準	個数	長さ	面積	体積	質量
重み(割合)	$\dfrac{n(L)}{\int_0^\infty n(L)\mathrm{d}L}$	$\dfrac{Ln(L)}{\int_0^\infty Ln(L)\mathrm{d}L}$	$\dfrac{\phi_S L^2 n(L)}{\phi_S \int_0^\infty L^2 n(L)\mathrm{d}L}$	$\dfrac{\phi_V L^3 n(L)}{\phi_V \int_0^\infty L^3 n(L)\mathrm{d}L}$	$\dfrac{\phi_V \rho_c L^3 n(L)}{\phi_V \rho_c \int_0^\infty L^3 n(L)\mathrm{d}L}$
加重平均径	$L_{1,0} = \dfrac{\int_0^\infty Ln(L)\mathrm{d}L}{\int_0^\infty n(L)\mathrm{d}L}$ (個数平均径)	$L_{2,1} = \dfrac{\int_0^\infty L^2 n(L)\mathrm{d}L}{\int_0^\infty Ln(L)\mathrm{d}L}$ (長さ平均径)	$L_{3,2} = \dfrac{\int_0^\infty L^3 n(L)\mathrm{d}L}{\int_0^\infty L^2 n(L)\mathrm{d}L}$ (面積平均径)	$L_{4,3} = \dfrac{\int_0^\infty L^4 n(L)\mathrm{d}L}{\int_0^\infty L^3 n(L)\mathrm{d}L}$ (体積平均径)	

(個数平均径) $$L_{1,0} = \int_0^\infty Ln(L)\mathrm{d}L \Big/ \int_0^\infty n(L)\mathrm{d}L = \frac{m_1}{m_0} = \frac{n_0(G\tau)^2}{n_0 G\tau} = G\tau$$

(長さ平均径) $$L_{2,1} = \int_0^\infty L^2 n(L)\mathrm{d}L \Big/ \int_0^\infty Ln(L)\mathrm{d}L = \frac{m_2}{m_1} = \frac{2n_0(G\tau)^3}{n_0(G\tau)^2} = 2G\tau$$

(面積平均径) $$L_{3,2} = \int_0^\infty L^3 n(L)\mathrm{d}L \Big/ \int_0^\infty L^2 n(L)\mathrm{d}L = \frac{m_3}{m_2} = \frac{6n_0(G\tau)^4}{2n_0(G\tau)^3} = 3G\tau$$

(体積平均径) $$L_{4,3} = \int_0^\infty L^4 n(L)\mathrm{d}L \Big/ \int_0^\infty L^3 n(L)\mathrm{d}L = \frac{m_4}{m_3} = \frac{24n_0(G\tau)^5}{6n_0(G\tau)^4} = 4G\tau$$

$$L_{j+1,j} = \frac{(j+1)!n_0(G\tau)^{j+2}}{j!n_0(G\tau)^{j+1}} = (j+1)G\tau$$ 一般式

（個数平均径）　$L_{1,0} = G\tau$

（長さ平均径）　$L_{2,1} = 2G\tau$

（面積平均径）　$L_{3,2} = 3G\tau$

（体積平均径）　$L_{4,3} = 4G\tau$

●個数基準　・・・小粒径粒子（数十ミクロン以下）

●長さ基準　・・・アスペクト比の高い粒子、針状粒子

●表面積基準・・・形状の複雑な粒子、比表面積の大きい粒子

●体積基準　・・・大粒径粒子（数百ミクロン～ミリサイズ）

標準偏差のモーメント

$$\sigma_0 \equiv \sqrt{\frac{\int_0^\infty (L-\bar{L})^2 n(L)\mathrm{d}L}{\int_0^\infty n(L)\mathrm{d}L}}$$

個数基準の標準偏差

標準偏差の定義式

$$\sigma \equiv \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}}$$

$$\sigma_0 = \sqrt{\frac{\int_0^\infty (L^2 - 2L\bar{L} + \bar{L}^2) n(L)\mathrm{d}L}{\int_0^\infty n(L)\mathrm{d}L}}$$

$$\sigma_0 = \sqrt{\frac{m_2 - 2m_1(m_1/m_0) + m_0(m_1/m_0)^2}{m_0}}$$

$$\left[\bar{L} \equiv L_{1,0} = \frac{m_1}{m_0}\right]$$

$$\sigma_0 = \sqrt{\frac{m_2 - 2m_1^2/m_0 + m_1^2/m_0}{m_0}}$$

$$\sigma_0 = \sqrt{(m_2/m_0) - (m_1/m_0)^2}$$

$$\left[\begin{aligned} &= \sqrt{2n_0(G\tau)^3/n_0G\tau - (n_0(G\tau)^2/n_0G\tau)^2} \\ &= \sqrt{2(G\tau)^2 - (G\tau)^2} = G\tau \end{aligned}\right]$$

$$\sigma_1 \equiv \sqrt{\frac{\int_0^\infty (L-\overline{L})^2 Ln(L)\mathrm{d}L}{\int_0^\infty Ln(L)\mathrm{d}L}}$$

長さ基準の標準偏差

$$\sigma_1 = \sqrt{\frac{\int_0^\infty (L^3 - 2L^2\overline{L} + L\overline{L}^2)n(L)\mathrm{d}L}{\int_0^\infty Ln(L)\mathrm{d}L}}$$

$$\sigma_1 = \sqrt{\frac{m_3 - 2m_2(m_2/m_1) + m_1(m_2/m_1)^2}{m_1}} \qquad \left[\overline{L} \equiv L_{2,1} = \frac{m_2}{m_1}\right]$$

$$\sigma_1 = \sqrt{\frac{m_3 - 2m_2{}^2/m_1 + m_2{}^2/m_1}{m_1}}$$

$$\boxed{\sigma_1 = \sqrt{(m_3/m_1) - (m_2/m_1)^2}}$$

$$\left[\begin{aligned} &= \sqrt{6n_0(G\tau)^4/n_0(G\tau)^2 - [2n_0(G\tau)^3/n_0(G\tau)^2]^2} \\ &= \sqrt{6(G\tau)^2 - 4(G\tau)^2} = \sqrt{2}G\tau \end{aligned}\right]$$

$$\sigma_2 \equiv \sqrt{\frac{\int_0^\infty (L-\overline{L})^2 \phi_S L^2 n(L)\mathrm{d}L}{\int_0^\infty \phi_S L^2 n(L)\mathrm{d}L}}$$

表面積基準の標準偏差

$$\sigma_2 = \sqrt{\frac{\int_0^\infty (L^4 - 2L^3\overline{L} + L^2\overline{L}^2)n(L)\mathrm{d}L}{\int_0^\infty L^2 n(L)\mathrm{d}L}}$$

$$\sigma_2 = \sqrt{\frac{m_4 - 2m_3(m_3/m_2) + m_2(m_3/m_2)^2}{m_2}} \qquad \left[\overline{L} \equiv L_{3,2} = \frac{m_3}{m_2}\right]$$

$$\sigma_2 = \sqrt{\frac{m_4 - 2m_3{}^2/m_2 + m_3{}^2/m_2}{m_2}}$$

$$\boxed{\sigma_2 = \sqrt{(m_4/m_2) - (m_3/m_2)^2}}$$

$$\left[= \sqrt{\begin{aligned}&24n_0(G\tau)^5/2n_0(G\tau)^3\\ &-[6n_0(G\tau)^4/2n_0(G\tau)^3]^2\end{aligned}} \right.$$

$$\left. = \sqrt{12(G\tau)^2 - 9(G\tau)^2} = \sqrt{3}G\tau \right]$$

$$\sigma_3 \equiv \sqrt{\frac{\int_0^\infty (L-\overline{L})^2 \phi_V L^3 n(L)\mathrm{d}L}{\int_0^\infty \phi_V L^3 n(L)\mathrm{d}L}}$$

体積基準の標準偏差

$$\sigma_3 = \sqrt{\frac{\int_0^\infty (L^5 - 2L^4\overline{L} + L^3\overline{L}^2) n(L)\mathrm{d}L}{\int_0^\infty L^3 n(L)\mathrm{d}L}}$$

$$\sigma_3 = \sqrt{\frac{m_5 - 2m_4(m_4/m_3) + m_3(m_4/m_3)^2}{m_3}} \qquad \left[\overline{L} \equiv L_{4,3} = \frac{m_4}{m_3}\right]$$

$$\sigma_3 = \sqrt{\frac{m_5 - 2m_4{}^2/m_3 + m_4{}^2/m_3}{m_3}}$$

$$\boxed{\sigma_3 = \sqrt{(m_5/m_3) - (m_4/m_3)^2}}$$

$$\left[\begin{aligned} &= \sqrt{120n_0(G\tau)^6/6n_0(G\tau)^4 - [24n_0(G\tau)^5/6n_0(G\tau)^4]^2} \\ &= \sqrt{20(G\tau)^2 - 16(G\tau)^2} = 2G\tau \end{aligned}\right]$$

$$\sigma_j = \sqrt{(m_{j+2}/m_j) - (m_{j+1}/m_j)^2}$$ 一般式

$$\sigma_j = \sqrt{\frac{(j+2)!n_0(G\tau)^{j+3}}{j!n_0(G\tau)^{j+1}} - \left[\frac{(j+1)!n_0(G\tau)^{j+2}}{j!n_0(G\tau)^{j+1}}\right]^2}$$ $\left[m_j \equiv j!n_0(G\tau)^{j+1}\right]$

$$\sigma_j = \sqrt{(j+2)(j+1)(G\tau)^2 - (j+1)^2(G\tau)^2}$$

$$\sigma_j = \sqrt{(j^2+3j+2)-(j^2+2j+1)}G\tau$$

$$\sigma_j = \sqrt{j+1}G\tau$$ 一般式

変動係数のモーメント

$$CV_j \equiv \frac{\sigma_j}{L_{j+1,j}}$$ *j*次の変動係数

$$CV_j = \frac{\sqrt{(m_{j+2}/m_j)-(m_{j+1}/m_j)^2}}{m_{j+1}/m_j} \quad \left[\begin{array}{l} \sigma_j \equiv \sqrt{(m_{j+2}/m_j)-(m_{j+1}/m_j)^2} \\ L_{j+1,j} \equiv m_{j+1}/m_j \end{array}\right]$$

$$CV_j = \frac{\sqrt{j+1}G\tau}{(j+1)G\tau} \quad \left[\begin{array}{l} \sigma_j \equiv \sqrt{j+1}G\tau \\ L_{j+1,j} \equiv (j+1)G\tau \end{array}\right]$$

$$CV_j = \frac{1}{\sqrt{j+1}}$$ 一般式

*CV*値が小さいほど単分散性良好（粒径がよく揃っている）
※*CV*20%以下で単分散

（個数基準）	$CV_0 = 1(100\%)$
（長さ基準）	$CV_1 = 1/\sqrt{2}\,(70.7\%)$
（表面積基準）	$CV_2 = 1/\sqrt{3}\,(57.7\%)$
（体積基準）	$CV_3 = 1/2\,(50\%)$

積算ふるい下分布のモーメント

(ア)個数基準の場合

$$U_0(L) \equiv \int_0^L n(L)\mathrm{d}L \Big/ \int_0^\infty n(L)\mathrm{d}L$$ (代表粒径L以下の全個数)÷(全個数)

$$U_0(L) = \int_0^L n_0 \exp\left(-\frac{L}{G\tau}\right)\mathrm{d}L \Big/ m_0 \quad \left[n(L) \equiv n_0 \exp\left(-\frac{L}{G\tau}\right), m_j = \int_0^\infty L^j n(L)\mathrm{d}L \right]$$

$$U_0(X) = \int_0^X n_0 \exp(-X)(G\tau)\mathrm{d}X \Big/ n_0 G\tau$$

変数変換

$$\left[\begin{aligned} & X \equiv L/G\tau, L = G\tau X \\ & \mathrm{d}X/\mathrm{d}L = 1/(G\tau),\ \mathrm{d}L = (G\tau)\mathrm{d}X \\ & m_j \equiv j! n_0 (G\tau)^{j+1} \end{aligned} \right]$$

$$U_0(X) = \frac{n_0 G\tau}{n_0 G\tau}\left[-\exp(-X)\right]_0^X$$

$$\boxed{U_0(X) = 1 - \exp(-X)}$$

$$0.5 = 1 - \exp(-X_{0,50})$$

$$X_{0,50} = 0.693$$

$$\boxed{L_{0,50} = 0.693 G\tau}$$ 個数基準のメジアン径

(イ)長さ基準の場合

$$U_1(L) \equiv \int_0^L Ln(L)\mathrm{d}L \Big/ \int_0^\infty Ln(L)\mathrm{d}L$$ (代表粒径L以下の全長)÷(全長)

$$U_1(L) = \int_0^L Ln_0 \exp\left(-\frac{L}{G\tau}\right)\mathrm{d}L \Big/ m_1 \quad \left[n(L) \equiv n_0 \exp\left(-\frac{L}{G\tau}\right), m_j = \int_0^\infty L^j n(L)\mathrm{d}L \right]$$

$$U_1(X) = n_0 (G\tau)^2 \int_0^X X\exp(-X)\mathrm{d}X \Big/ n_0 (G\tau)^2$$ 変数変換($X=L/G\tau$)

$$U_1(X) = \frac{n_0 (G\tau)^2}{n_0 (G\tau)^2}\left[[-X\exp(-X)]_0^X - \int_0^X 1\cdot[-\exp(-X)]\mathrm{d}X \right]$$

部分積分法

$f(X)=X,\ g'(X)=\exp(-X)$と置く

$$U_1(X) = -X\exp(-X) + 1 - \exp(-X)$$

$$U_1(X) = 1-(1+X)\exp(-X)$$

$$0.5 = 1-(1+X_{1,50})\exp(-X_{1,50})$$

$$X_{1,50} = 1.68$$

$$L_{1,50} = 1.68G\tau$$

長さ基準のメジアン径

※公式の導出(合成関数の微分より)

$$[f(X)g(X)]' = f'(X)g(X) + f(X)g'(X)$$

$$f(X)g'(X) = [f(X)g(X)]' - f'(X)g(X)$$

$$\int_0^\infty f(X)g'(X)\mathrm{d}X = [f(X)g(X)]_0^\infty - \int_0^\infty f'(X)g(X)\mathrm{d}X$$

（ウ）表面積基準の場合

（代表粒径L以下の全表面積）÷（全表面積）

$$U_2(L) \equiv \int_0^L \phi_S L^2 n(L) \mathrm{d}L \Big/ \int_0^\infty \phi_S L^2 n(L) \mathrm{d}L = \int_0^L L^2 n(L) \mathrm{d}L \Big/ \int_0^\infty L^2 n(L) \mathrm{d}L$$

$$U_2(L) = \int_0^L L^2 n_0 \exp\left(-\frac{L}{G\tau}\right) \mathrm{d}L \Big/ m_2 \quad \left[n(L) \equiv n_0 \exp\left(-\frac{L}{G\tau}\right), m_j = \int_0^\infty L^j n(L) \mathrm{d}L \right]$$

$$U_2(X) = n_0 (G\tau)^3 \int_0^X X^2 \exp(-X) \mathrm{d}X \Big/ 2 n_0 (G\tau)^3$$

変数変換(X=$L/G\tau$)

$$U_2(X) = \frac{n_0 (G\tau)^3}{2 n_0 (G\tau)^3} \left[\left[-X^2 \exp(-X) \right]_0^X - \int_0^X 2X[-\exp(-X)] \mathrm{d}X \right]$$

$$U_2(X) = \left[-X^2 \exp(-X) + 2 - 2(1+X)\exp(-X) \right] \Big/ 2$$

$f(X)=X^2$, $g'(X)=\exp(-X)$

$$U_2(X) = \left[2 - (2 + 2X + X^2)\exp(-X) \right] \Big/ 2$$

$$\left[\int_0^X X \exp(-X) \mathrm{d}X = 1 - (1+X)\exp(-X) \right]$$

$$U_2(X) = 1 - (1 + X + X^2/2)\exp(-X)$$

$$0.5 = 1 - (1 + X_{2,50} + X_{2,50}{}^2/2)\exp(-X_{2,50}) \longrightarrow X_{2,50} = 2.67$$

$$L_{2,50} = 2.67 G\tau$$

表面積基準のメジアン径

（エ）質量基準の場合

（代表粒径L以下の全質量）÷（全質量）

$$U_3(L) \equiv \int_0^L \phi_V \rho_c L^3 n(L)\mathrm{d}L \Big/ \int_0^\infty \phi_V \rho_c L^3 n(L)\mathrm{d}L = \int_0^L L^3 n(L)\mathrm{d}L \Big/ \int_0^\infty L^3 n(L)\mathrm{d}L$$

$$U_3(L) = \int_0^L L^3 n_0 \exp\left(-\frac{L}{G\tau}\right)\mathrm{d}L \Big/ m_3 \quad \left[n(L) \equiv n_0 \exp\left(-\frac{L}{G\tau}\right), m_j = \int_0^\infty L^j n(L)\mathrm{d}L \right]$$

$$U_3(X) = n_0 (G\tau)^4 \int_0^X X^3 \exp(-X)\mathrm{d}X \Big/ 6n_0 (G\tau)^4$$

変数変換(X=L/$G\tau$)

$$U_3(X) = \frac{n_0 (G\tau)^4}{6n_0 (G\tau)^4} \left[\left[-X^3 \exp(-X) \right]_0^X - \int_0^X 3X^2 [-\exp(-X)]\mathrm{d}X \right]$$

$$U_3(X) = \left[-X^3 \exp(-X) + 3\left\{ 2 - (2 + 2X + X^2)\exp(-X) \right\} \right] \Big/ 6$$

$f(X)=X^3$

$g'(X)=\exp(-X)$

$$U_3(X) = \left[6 - (6 + 6X + 3X^2 + X^3)\exp(-X) \right] \Big/ 6$$

$$\left[\int_0^X X^2 \exp(-X)\mathrm{d}X = 2 - (2 + 2X + X^2)\exp(-X) \right]$$

$$\boxed{U_3(X) = 1 - (1 + X + X^2/2 + X^3/6)\exp(-X)}$$

$$0.5 = 1 - (1 + X_{3,50} + X_{3,50}{}^2/2 + X_{3,50}{}^3/6)\exp(-X_{3,50}) \longrightarrow X_{3,50} = 3.67$$

$$\boxed{L_{3,50} = 3.67G\tau}$$

質量基準のメジアン径

※まとめ（積算ふるい下分布、メジアン径）

（個数基準）
$$\begin{cases} U_0(X) = 1 - \exp(-X) \\ L_{0,50} = 0.693G\tau \end{cases}$$

（長さ基準）
$$\begin{cases} U_1(X) = 1 - (1 + X)\exp(-X) \\ L_{1,50} = 1.68G\tau \end{cases}$$

（表面積基準）
$$\begin{cases} U_2(X) = 1 - (1 + X + X^2/2)\exp(-X) \\ L_{2,50} = 2.67G\tau \end{cases}$$

（質量基準）
$$\begin{cases} U_3(X) = 1 - (1 + X + X^2/2 + X^3/6)\exp(-X) \\ L_{3,50} = 3.67G\tau \end{cases}$$

一般式

$$U_j(X) = 1 - \sum \frac{X^j}{j!}\exp(-X)$$

積算ふるい上分布のモーメント

一般式

$$R_j(X) \equiv 1 - U_j(X) = \sum \frac{X^j}{j!} \exp(-X)$$

（個数基準） $R_0(X) = \exp(-X)$

（長さ基準） $R_1(X) = (1 + X)\exp(-X)$

（表面積基準） $R_2(X) = (1 + X + X^2/2)\exp(-X)$

（質量基準） $R_3(X) = (1 + X + X^2/2 + X^3/6)\exp(-X)$

頻度分布のモーメント

（ア）個数基準の場合

$$f_0(X) \equiv \frac{\mathrm{d}U_0(X)}{\mathrm{d}X} = \frac{\mathrm{d}}{\mathrm{d}X}[1-\exp(-X)]$$

$$\boxed{f_0(X) = \exp(-X)}$$

$$\frac{\mathrm{d}f_0(X)}{\mathrm{d}X} = \frac{\mathrm{d}}{\mathrm{d}X}[\exp(-X)] = -\exp(-X)$$

$$-\exp(-X_{0,\mathrm{m}}) = 0$$

分布の極大値を与える粒径がモード径なので
分布関数の微分値を0とおく

$$X_{0,\mathrm{m}} = \infty$$

$$\boxed{L_{0,\mathrm{m}} = \infty}$$

個数基準のモード径（発散）

（イ）長さ基準の場合

$$f_1(X) \equiv \frac{\mathrm{d}U_1(X)}{\mathrm{d}X} = \frac{\mathrm{d}}{\mathrm{d}X}[1-(1+X)\exp(-X)]$$

$$f_1(X) = 0 - \left[\exp(-X)\frac{\mathrm{d}}{\mathrm{d}X}(1+X) + (1+X)\frac{\mathrm{d}}{\mathrm{d}X}\exp(-X)\right]$$

$$f_1(X) = 0 - \left[\exp(-X) - (1+X)\exp(-X)\right]$$

$$\boxed{f_1(X) = X\exp(-X)}$$

$$\frac{\mathrm{d}f_1(X)}{\mathrm{d}X} = \frac{\mathrm{d}}{\mathrm{d}X}[X\exp(-X)] = \exp(-X) - X\exp(-X) = (1-X)\exp(-X)$$

$$(1-X_{1,\mathrm{m}})\exp(-X_{1,\mathrm{m}})=0$$

分布の極大値を与える粒径がモード径なので分布関数の微分値を0とおく

$$X_{1,\mathrm{m}} = 1$$

$$\boxed{L_{1,\mathrm{m}} = G\tau}$$

長さ基準のモード径

(ウ)表面積基準の場合

$$f_2(X) \equiv \frac{\mathrm{d}U_2(X)}{\mathrm{d}X} = \frac{\mathrm{d}}{\mathrm{d}X}[1-(1+X+X^2/2)\exp(-X)]$$

$$f_2(X) = 0 - \left[\exp(-X)\frac{\mathrm{d}}{\mathrm{d}X}(1+X+X^2/2) + (1+X+X^2/2)\frac{\mathrm{d}}{\mathrm{d}X}\exp(-X)\right]$$

$$f_2(X) = 0 - \left[\exp(-X)(1+X) - (1+X+X^2/2)\exp(-X)\right]$$

$$\boxed{f_2(X) = (X^2/2)\exp(-X)}$$

$$\frac{\mathrm{d}f_2(X)}{\mathrm{d}X} = \frac{\mathrm{d}}{\mathrm{d}X}[(X^2/2)\exp(-X)] = X\exp(-X) - (X^2/2)\exp(-X)$$

$$\frac{\mathrm{d}f_2(X)}{\mathrm{d}X} = [X-(X^2/2)]\exp(-X)$$

分布の極大値を与える粒径がモード径なので分布関数の微分値を0とおく

$$[X_{2,\mathrm{m}} - (X_{2,\mathrm{m}}{}^2/2)]\exp(-X_{2,\mathrm{m}})=0$$

$$X_{2,\mathrm{m}} = 2$$

$$\boxed{L_{2,\mathrm{m}} = 2G\tau}$$

表面積基準のモード径

(エ)質量基準の場合

$$f_3(X) \equiv \frac{\mathrm{d}U_3(X)}{\mathrm{d}X} = \frac{\mathrm{d}}{\mathrm{d}X}[1-(1+X+X^2/2+X^3/6)\exp(-X)]$$

$$f_3(X) = 0 - \left[\begin{array}{l} \exp(-X)\dfrac{\mathrm{d}}{\mathrm{d}X}(1+X+X^2/2+X^3/6) \\ +(1+X+X^2/2+X^3/6)\dfrac{\mathrm{d}}{\mathrm{d}X}\exp(-X) \end{array}\right]$$

$$f_3(X) = 0 - \left[\exp(-X)(1+X+X^2/2) - (1+X+X^2/2+X^3/6)\exp(-X)\right]$$

$$\boxed{f_3(X) = (X^3/6)\exp(-X)}$$

$$\frac{\mathrm{d}f_3(X)}{\mathrm{d}X} = \frac{\mathrm{d}}{\mathrm{d}X}[(X^3/6)\exp(-X)] = (X^2/2)\exp(-X) - (X^3/6)\exp(-X)$$

$$[(X_{3,\mathrm{m}}{}^2/2) - (X_{3,\mathrm{m}}{}^3/6)]\exp(-X_{3,\mathrm{m}}) = 0$$

分布の極大値を与える粒径がモード径なので分布関数の微分値を0とおく

$$X_{3,\mathrm{m}} = 3$$

$$\boxed{L_{3,\mathrm{m}} = 3G\tau}$$

質量基準のモード径

※まとめ（頻度分布、モード径）

（個数基準）
$$\begin{cases} f_0(X) = \exp(-X) \\ L_{0,\mathrm{m}} = \infty \end{cases}$$

（長さ基準）
$$\begin{cases} f_1(X) = X\exp(-X) \\ L_{1,\mathrm{m}} = G\tau \end{cases}$$

（表面積基準）
$$\begin{cases} f_2(X) = (X^2/2)\exp(-X) \\ L_{2,\mathrm{m}} = 2G\tau \end{cases}$$

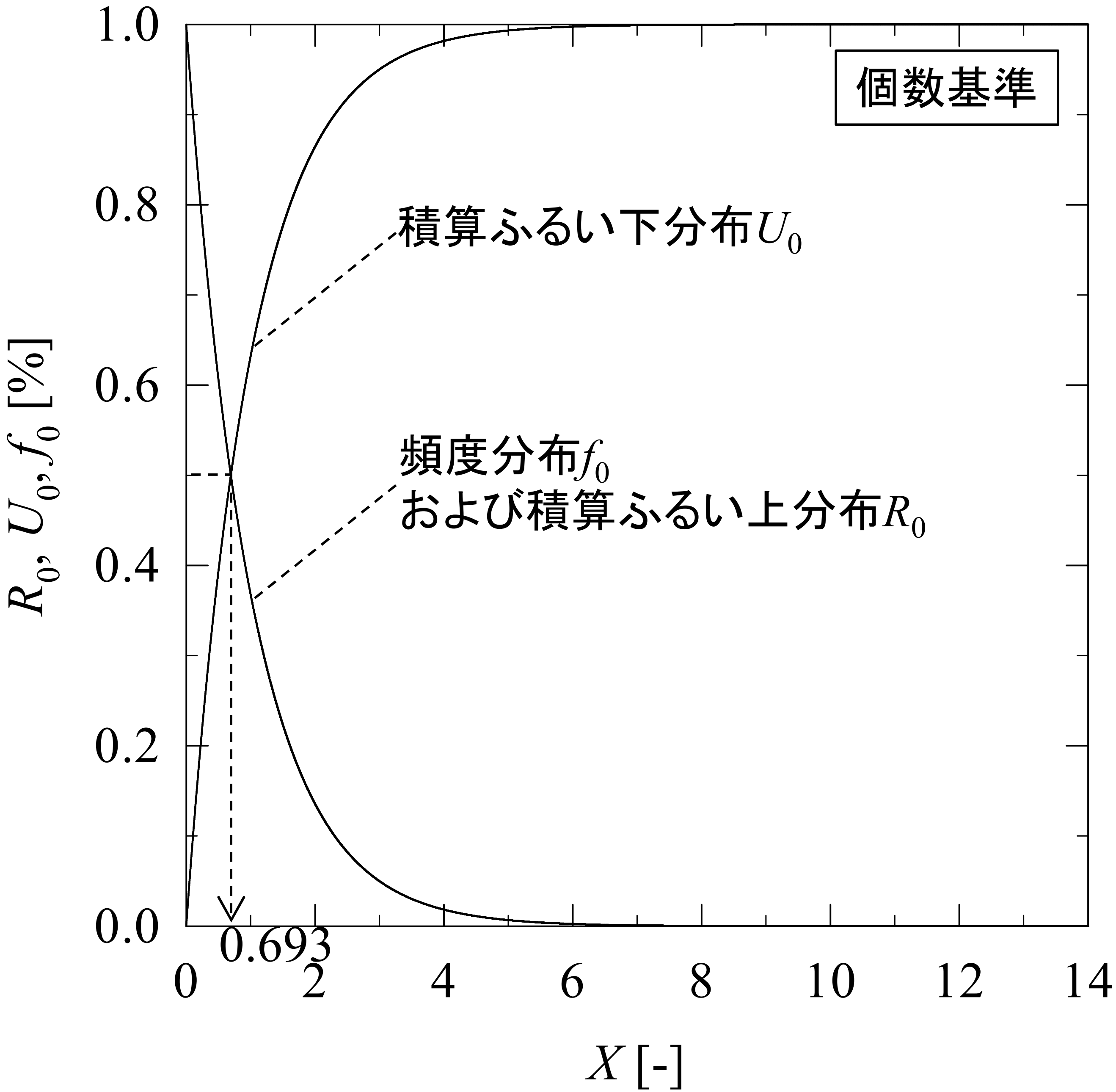
（質量基準）
$$\begin{cases} f_3(X) = (X^3/6)\exp(-X) \\ L_{3,\mathrm{m}} = 3G\tau \end{cases}$$

一般式

$$f_j(X) = \frac{X^j}{j!}\exp(-X)$$

$$L_{j,\mathrm{m}} = jG\tau \; (j \neq 0)$$

理論上の粒径分布（個数基準）



理論上の粒径分布（質量基準）